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GS-322

IV Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - IV

(CBCS) (F+R) (2015-16 & Onwards)

Time : 3 Hours

Max. Marks : 70

Instructions : Answer all Parts.

PART-A

1. Answer **any five** questions : **5x2=10**

(a) Define normal subgroup of a group.

(b) Verify whether $f : (Z, +) \to (Z, +)$ defined by $f(n) = 4n + 2$ is a homomorphism.

(c) Obtain half range sine series of $f(x) = 2x - 1$ in the interval (0, 1).

(d) Write the necessary conditions for $f(x, y)$ to have extreme values at (a, b).

(e) Find $L[\cos^2(2t)]$

(f) Find $L^{-1}\left[\dfrac{2S-1}{S^2+16}\right]$

(g) Solve $(D^2 + 4D + 2)y = 0$.

(h) Verify whether $(1-x^2)y'' - 3xy' - y = 0$ is exact.

PART-B

Answer **one** full question : **1x15=15**

2. (a) Prove that a subgroup H of a group G is a normal subgroup of G if and only if every left coset of H is also a right coset of H in G.

(b) If N is a normal subgroup of a group G and G/N is a collection of all cosets of N in G.

Prove that G/N is a group under the binary operation $(N_a)(N_b) = N_{ab}, \forall\ N_a, N_b \in G/N$

(c) If $f : G \to G$ is a homomorphism of a group G into itself and H is a cyclic subgroup of G then prove that f(H) is also a cyclic subgroup of G.

OR

3. (a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

(b) Let $G = \left\{\dfrac{a+b\sqrt{2}}{a, b \in Q}\right\}$. Show that $f : (G, +) \to (G, +)$ defined by $f(a+b\sqrt{2}) = a-b\sqrt{2}$ is an isomorphism.

(c) State and prove Cayley's theorem.

P.T.O.

PART-C

Answer **any two** full questions : **2x15=30**

4. (a) Obtain Fourier series expansion of the function $f(x)=\begin{cases} -x \text{ for } -\pi < x < 0 \\ x \text{ for } 0 < x < \pi \end{cases}$

(b) Obtain Fourier series expansion of the function $f(x)=x-x^2$ in the interval $(-1,1)$.

(c) Find Taylor's series expansion of $f(x, y)=\dfrac{y^2}{x^3}$ about the point $(1, -1)$ upto 2^{nd} degree terms.

OR

5. (a) Find the maximum and minimum distances of the point (1,2,3) from the sphere $x^2+y^2+z^2=56$ using Lagrange's method.

(b) Find the extreme values of the function $f(x, y)=x^3+y^3-3x-12y+20$

(c) Find the half range Fourier cosine series of $f(x)=\sin x$ in the interval $(0, \pi)$.

6. (a) (i) If $L[f(t)]=F(S)$ then prove that $L[f(at)]=\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$.

(ii) Find $L[e^{-t}(2\sin 2t.\ \cos 5t)]$

(b) Find the Laplace transform of the function

$$f(t)=\begin{cases} 1, & 0<t<T \\ -1, & T<t<2T \end{cases}$$ given that f(t) is periodic with period 2T.

(c) Find $L^{-1}\left[\dfrac{S+5}{(S-1)(S^2+4)}\right]$

OR

7. (a) If $L[f(t)]=F(S)$ then prove that $L\left[\dfrac{f(t)}{t}\right]=\int_S^\infty F(S)ds$. Hence evaluate $L\left[\dfrac{\sin t}{t}\right]$

(b) Find $L^{-1}\left[\log\left(\dfrac{S^2+1}{S(S+1)}\right)\right]$

(c) State convolution theorem and use it to find $L^{-1}\left[\dfrac{S}{(S^2+a^2)^2}\right]$

PART-D

1x15=15

Answer **one** full question :

8. (a) Solve $y'' + 3y' + 2y = \cos^2 x$

   (b) Solve $x^2 y'' - xy' + 2y = x \log x$

   (c) solve $xy'' - (2x+1)y' + (x+1)y = x^2 e^{2x}$ given that e^x is a part of the complementary function.

OR

9. (a) Solve $(D^2 - 2D + 4)y = e^x \cos x$

   (b) Solve $\frac{dx}{dt} + 7x - y = 0; \quad \frac{dy}{dt} + 2x + 5y = 0$

   (c) Solve $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$ using the transformation $z = x^2$.

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